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A METHOD FOR THE ANALYSIS OF GEOMAGNETIC VARIATION FIELD AND ITS APPLICATION TO THE YEAR-TO-YEAR CHANGE OF Sq ACTIVITY

By

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Abstract

A method to determine the main terms of spherical harmonic coefficients of the geomagnetic variation field is studied with reasonable approximations, under the condition of 'rotation-free'. The method is applied to the investigation of the year-to-year change of Sq activity in a period of the descendent stage of solar activity. The result shows that g_2^1, the coefficient of cosine-term, decreases with the declining of sunspot activity, meanwhile h_2^1, the coefficient of sine-term, has no significant correlation.

1. Introduction

The analysis of the geomagnetic Sq field has been made by many workers, especially by Chapman [1919], Benkova [1940], Hasegawa and Ota [1950], and recently by Price and Wilkins [1963], Matsushita and Maeda [1965], and Sugiura and Hagan [1967]. The methods of analysis used by these workers may be generally classified into the following two kinds ; say, the surface integral method and the method with the spherical harmonic coefficients. The former may be advantageous for the satisfaction of the condition of 'rotation-free', but it needs prudent consideration about the determination of the datum line, as well as the complicated technique of calculation. On the contrary, the latter bears the demerit that the values of coefficients obtained from the north component, X, and from the east component, Y, would often disaccord each other. For example, Hasegawa and Ota analyzed Sq field of the Second Polar Year by a method of graphical integration, and got the value of g_2^1 to be 10.1 from the north component and 7.1 from the east component. The present study is also based on the spherical harmonic coefficients fundamentally, yet the condition of 'rotation-free' is satisfied under some approximations in view of getting main terms of the harmonics.

2. Method of analysis

The geomagnetic coordinates are adopted in this paper through. The potential $V(\Theta, \Lambda, T)$ at a point on the surface of the earth at colatitude Θ and longitude Λ is expressed by a series of spherical harmonics as follows,

$$V = C + R\Sigma\Sigma\{g_n^m \cos m(T+\Lambda) + h_n^m \sin m(T+\Lambda)\}P_n^m(\Theta) \tag{1}$$

where C is an arbitrary const., R is the radius of the earth, $T+\Lambda$ denotes local time and P_n^m is expressed by Schmidt's functions.

In calculating the coefficients g_n^m and h_n^m particular considerations have been paid to the following points. First, the data used in this study are all derived from the yearly-mean of all days. Usually, the data of International Quiet Days or those of Ten Quiet Days are adopted on the analysis of Sq. In these cases, however, if the extent of disturbance differs in each month or season, its influence disproportionately affects upon pure Sq. Or, in other words, even in the case of selected days, variation field might not consist of pure Sq, therefore, disproportionate superposition of the disturbed field makes the statistical result obscure. By using the yearly-mean of all days the factor of disturbance should be expressed in the mean as well as that of Sq be so. This is very advantageous for one of the author's present purposes, that is, to get the relation between the sunspot activity and that of daily variation field by comparing the coefficients year by year. Use of all days has another merit of being free from the noncyclic change of the geomagnetic field which may be caused by magnetic storms, secular change and other origins.

Next, the bounds of data to be used is examined. The number of the observatories from which data is prepared for this study is some twenty each year, selection of which is made covering middle and low latitudes, and weighted those available through the period concerned. Coordinates of the observatories used are listed in Table 1. In many cases the data obtained from respective observatories are directly averaged on latitude by means of least-square-method, regardless of the regional or the local inequallity of variation. In the present study, as has been utilized by Hasegawa and Ota, and some other workers, distribution-maps of each Fourier-component of the magnetic force are prepared at first, then, by examining local anomalies the most reasonable smoothing of intensities is practised. Examples of these distribution-maps are shown in Figs. 1a–1d. As is seen in these figures, a fairly large inequallity of variation is noticed in the European zone. Some difficulty for smoothing in maps of $a_1(Y)$ and $b_1(X)$ may be produced by the influence of disturbance which might be a little stressed in the present case (all days). Now, the data at and near the equator are omitted on account of the following reasons. The variation in

Table 1. List of observatories used in this study

Observatory	Geographic		Geomagnetic	
	Lat. (ϕ)	Long. (λ)	Lat. (Φ)	Long. (Λ)
Sitka	N57°.04′	W135°.20′	60°.0	275°.4
Lovo	N59.21	E017.50	58.1	105.8
Agincourt	N43.47	W079.16	55.2	346.9
Victoria	N48.31	W123.25	54.3	292.7
Fredericksburg	N38.21	W077.22	49.6	349.8
Wien-Kobenzl	N48.16	E016.19	47.9	097.8
Logrono	N42.51	W002.28	46.1	077.2
San Fernando	N36.28	W006.12	41.0	071.4
Tucson	N32.15	W110.50	40.4	312.2
Memambetsu	N43.55	E114.12	34.0	208.5
San Juan	N18.23	W066.07	29.9	003.2
Kakioka	N36.14	E140.11	26.0	206.1
Aso	N32.53	E131.01	22.1	198.1
M'Bour	N14.24	W016.58	21.3	055.0
Honolulu	N21.18	W158.06	21.1	266.5
Kanoya	N31.25	E130.53	20.5	198.2
Paramaribo	N05.50	W055.10	17.0	014.5
Moca	N03.21	E008.40	05.7	078.6
Addis Ababa	N09.02	E038.46	05.3	109.2
Guam	N13.35	E144.52	04.0	212.9
Jarvis Is.	S00.23	W160.02	−00.6	106.5
Huancayo	S12.02	W075.20	−00.6	353.8
Koror	N07.20	E134.30	−03.2	203.4
Luanda	S08.55	E013.10	−07.2	080.6
Hollandia	S02.34	E140.31	−12.6	210.3
Apia	S13.48	W171.46	−16.0	260.2
Tananarive	S18.55	E047.33	−23.1	112.1
Trelew	S43.15	W065.19	−32.3	003.2
Hermanus	S34.25	E019.14	−33.7	081.7
Amberley	S43.09	E172.43	−47.7	252.5

very low latitude, where the equatorial electrojet affects severely upon the geomagnetic field, behaves much complicatedly in connection with the relative situation of geographic, geomagnetic and dip equators. And granted that the variation in this zone is normalized somehow, the sharp latitudinal change might not meet such an approximation as considered bellow. Furthermore, the expression by Schmidt's function, with which the present study is going to develop, shows that the accuracy of determining the coefficients reduces according as data come to low latitude. The data in high latitudes are also excepted to avoid the overcoming of the polar disturbance. Thus, the data handled in

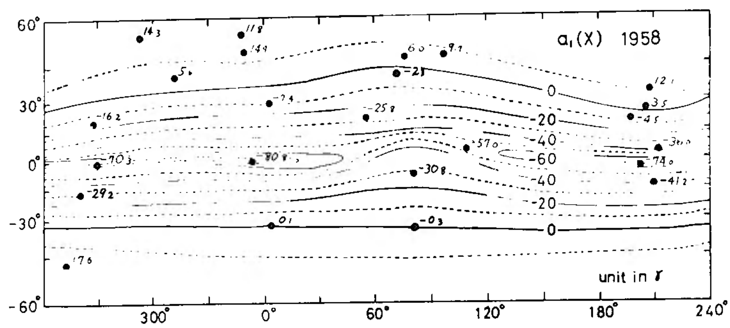

Fig. 1a

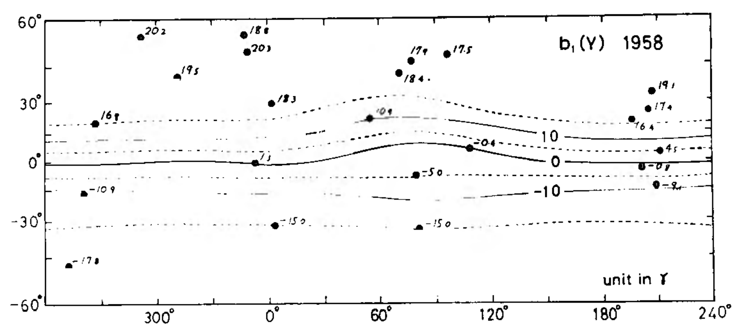

Fig. 1b

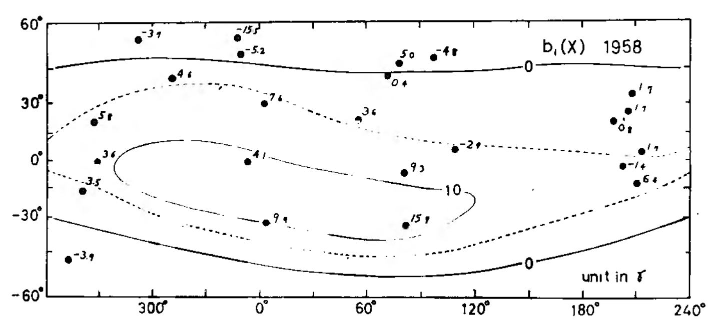

Fig. 1c

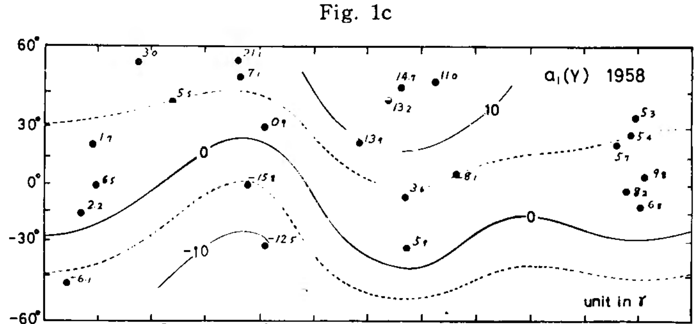

Fig. 1d

Fig. 1a–1d. Examples of distribution-maps of each Fourier-component of the variation field in the geomagnetic coordinates.

this study are finally limitted between 15° and 45° in latitude.

Hence, becomming free from the sharp change, the higher terms of the spherical function may be reasonably neglected. Utilizing the approximations based on the reason mentioned above, these coefficients are determined as follows.

By neglecting higher terms than the third, and considering only even terms ($X_2^1\cdots$, $Y_2^1\cdots$), the north and the east components of the variation field are expressed by the following equations.

$$a_1(X)=g_2^1X_2^1(\Theta)+g_4^1X_4^1(\Theta)+g_6^1X_6^1(\Theta) \tag{2a}$$

$$b_1(Y)=g_2^1Y_2^1(\Theta)+g_4^1Y_4^1(\Theta)+g_6^1Y_6^1(\Theta) \tag{2b}$$

$$b_1(X)=h_2^1X_2^1(\Theta)+h_4^1X_4^1(\Theta)+h_6^1X_6^1(\Theta) \tag{2c}$$

$$a_1(Y)=h_2^1Y_2^1(\Theta)+h_4^1Y_4^1(\Theta)+h_6^1Y_6^1(\Theta) \tag{2d}$$

where $a_1(X)$, $b_1(Y)$, $b_1(X)$ and $a_1(Y)$ are the Fourier-components of magnetic force, and X_n^m and Y_n^m are functions defined by $X_n^m=\frac{1}{n}\frac{dP_n^m}{d\Theta}$, $Y_n^m=\frac{m}{n}\frac{P_n^m}{\sin\Theta}$.

First, g_6^1 is preliminarily determined from equation (2a) by graphical method utilizing the relation between X_2^1, X_4^1 and X_6^1. Then, applying g_6^1 thus determined to equations (2a) and (2b), g_2^1 and g_4^1 are obtained at the latitudes of every 5°. h_2^1 and h_4^1 are also determined from equations (2c) and (2d) in the same way. As these coefficients should naturally be independent of latitude, the coefficients thus obtained provisionally are re-examined by successive approxi-

Table 2. Values of spherical harmonic coefficients g_2^1, g_4^1, h_2^1 and h_4^1 obtained from the data at each latitude (Force unit=γ)

Coeff.	Φ	1958	1959	1960	1961	1962	Coeff.	Φ	1958	1959	1960	1961	1962
g_2^1	45°	15.6	13.1	11.3	8.4	7.4	h_2^1	45°	−5.6	−5.7	−6.3	−6.0	−5.3
	40	15.9	13.9	11.5	8.5	7.7		40	−5.3	−5.8	−6.3	−6.0	−5.2
	35	16.3	14.3	12.2	9.1	7.9		35	−5.3	−5.8	−6.3	−6.0	−5.1
	30	16.5	14.3	12.5	9.4	8.0		30	−5.6	−5.4	−6.4	−5.8	−5.1
	25	16.4	14.4	12.6	9.4	7.7		25	−5.5	−5.2	−6.3	−5.5	−5.2
	20	16.4	14.2	12.6	9.0	7.3		20	−5.3	−5.1	−6.0	−5.3	−5.1
	15	16.3	14.2	12.5	9.3	7.7		15	−5.2	−5.0	−6.0	−5.2	−5.0
	mean	16.2	14.1	12.2	9.0	7.7		mean	−5.4	−5.4	−6.2	−5.7	−5.1
g_4^1	45°	−4.0	−3.7	−2.8	−2.2	−2.2	h_4^1	45°	−0.3	−0.3	−0.8	−0.7	−0.5
	40	−3.6	−3.9	−2.7	−2.1	−2.2		40	−0.4	−0.3	−1.0	−0.7	−0.6
	35	−3.9	−3.9	−2.6	−2.2	−2.2		35	−0.4	−0.5	−1.0	−0.6	−0.6
	30	−3.9	−4.2	−2.6	−2.2	−2.7		30	−0.5	−0.4	−1.0	−0.6	−0.6
	25	−4.6	−4.6	−2.9	−2.5	−2.9		25	−0.5	−0.4	−0.9	−0.5	−0.6
	20	−4.2	−4.6	−2.9	−2.6	−3.1		20	−0.5	−0.4	−0.9	−0.5	−0.6
	15	−4.4	−4.6	−2.9	−2.4	−2.8		15	−0.4	−0.4	−0.9	−0.5	−0.6
	mean	−4.1	−4.2	−2.8	−2.3	−2.6		mean	−0.4	−0.4	−0.9	−0.6	−0.6

mation, although its extent should be bounded by the observational values. The results obtained through such processes for the year from 1958 to 1962 are shown in Table 2, and final values of the coefficients are given by an average over latitudes.

3. Year-to-year change of the activity of daily variation

In Fig. 2 is shown the relation between the coefficients of the diurnal term thus obtained and the relative sunspot numbers during the period from 1958 to 1962 (the descendent stage of solar activity). This figure indicates that g_2^1, which is the coefficient of cosine-term, decreases year by year, in good accordance with the tendency of the sunspot activity, and that h_2^1, which is the coefficient of sine-term, does not show monotonically decreasing curve like g_2^1, or rather seems to keep nearly the same value through the period (if remotely compared, slowly decreases after the maximum at 1960). Meanwhile g_4^1 also seems to decrease with sunspot activity, though its tendency is not so distinct as that of g_2^1. As for h_4^1, it is somewhat difficult to notice any systematic change owing to its small value. For the comparison, values of the am-

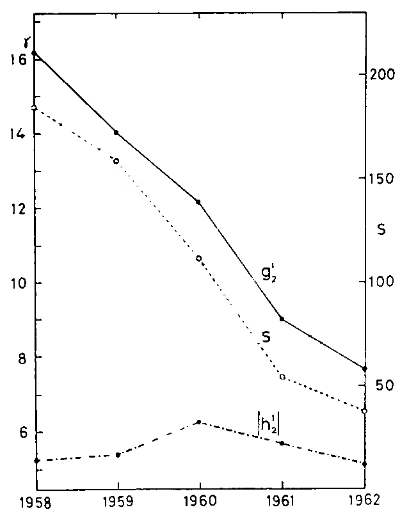

Fig. 2. Relation between the spherical harmonic coefficients g_2^1, h_2^1, and the yearly mean of relative sunspot number S.

Table 3 Comparison of amplitudes and phases determined by several workers (Schmidt-normalized, force unit$=\gamma$)

Year	Sunspot number	C_2^1	α_2^1	
1958	185	17.1	18°	
1959	159	15.1	21	
1960	112	13.7	27	Yasuhara [1967]
1961	54	10.7	32	
1962	38	9.2	34	
1902	5	7.0	35	Chapman [1919]
1905	64	10.1	24	
1932–33	8	10.5	14	Hasegawa and Ota [1950]
1958	185	19.5	2	Matsushita and Maeda [1965]
(1958 June and July)	130	14.2	10	Sugiura and Hagan [1967]

plitude C_2^1 and phase angle α_2^1 are listed in Table 3 together with the data by several other workers.

4. Discussion and conclusion

Some consideration should be paid about the results of the present analysis. The values of h_2^1 and the sunspot numbers are in a good linear correlation, although there remains a little scruple until supported by the data at the epoch of very small suspot number. Now, it seems to be unaccountable that the tendency of h_1^2 does not follow that of the sunspot activity. But, an interpretation may be given by introducing the result by Ota [1954]. Accoring to his analysis, which treated the data of one year in the Second Polar Year, the amplitudes of sine-term increase in the order of the cases for quiet days, all days and disturbed days, and those of cosine-term, on the contrary, seems to keep almost the same value in any case, as is clear in Figs. 3a-3b. As sine-

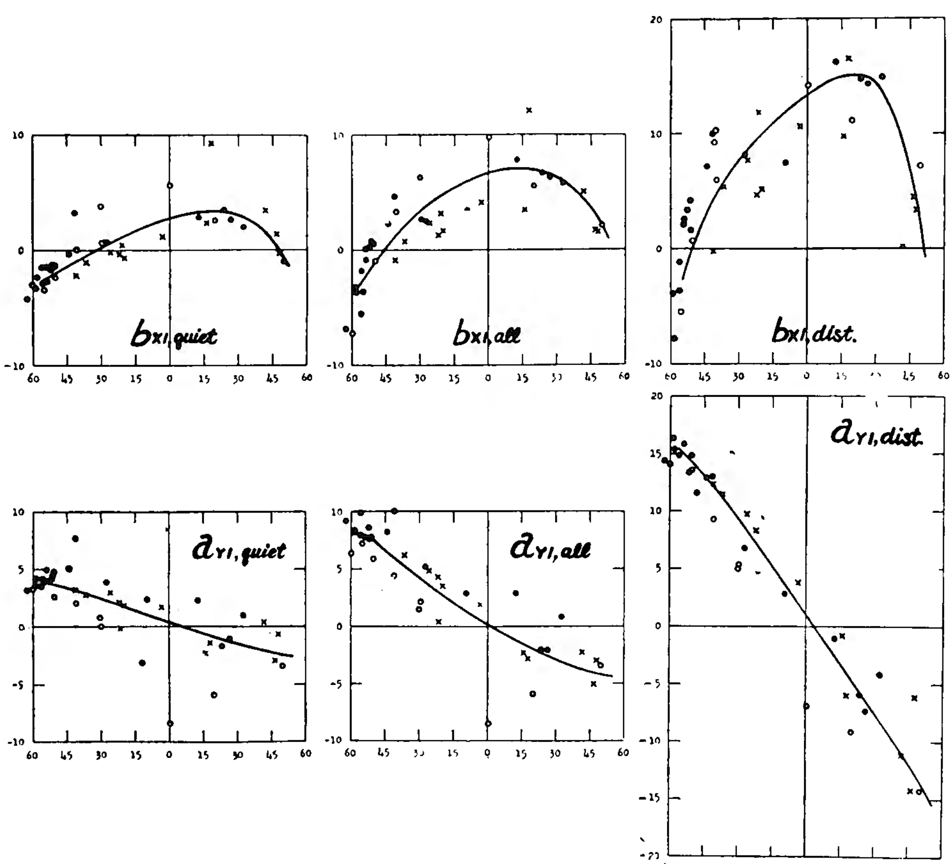

Fig. 3a.

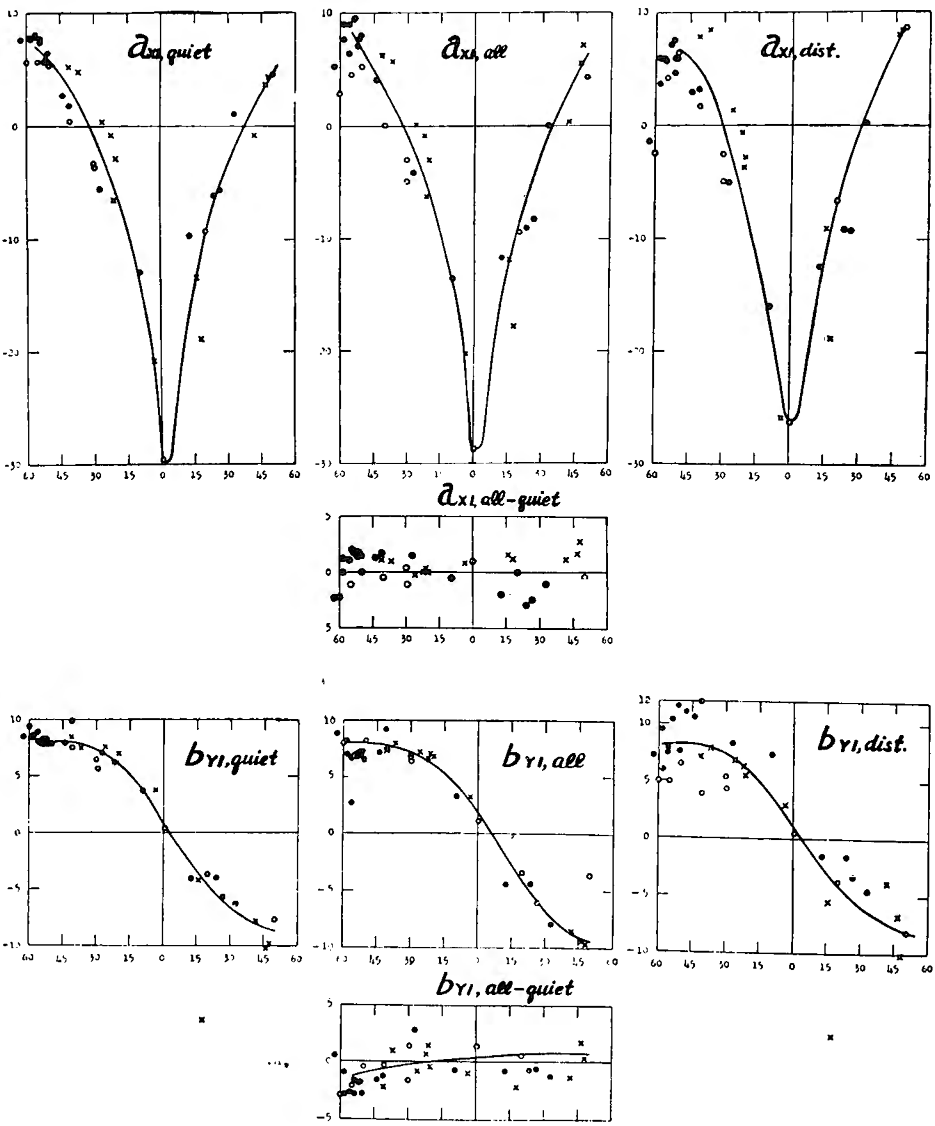

Fig. 3b.

Figs. 3a–3b. Latitudinal distributions of each Fourier-component of the variation field in the Second Polar Year. Fig. 3a corresponds to since-term, and Fig. 3b to cosine-term. Force unit $=\gamma$ (After Ota [1954]).

term is regarded as a measure of the polar disturbance, the result may be well convincing on this part, while the constancy of the amplitudes of cosine-term, which may denotes the pure Sq, through all three cases implies that there is no direct connection between the activity of Sq and that of disturbance.

Now, in Table 3, the phase angles of the present case seem to be relatively larger compared with those by other workers. However, this fact may also be explained as the effect of the increase of sine-term based on the data of all days.

Although some approximations are introduced in the process of calculation, the following features of this method should be emphasized. They are: the potential function is essentially satisfied by solving the simultaneous equations of both X and Y components of the force at each latitude, being free from the consideration about the datum-line, so far as *amplitude* is concerned. This method is to be extended so that few magnetic data selected in middle latitudes may afford to determine some main harmonics of the Sq potential in a fairly good approximation.

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References

Benkova, N. P., 1940; Spherical harmonic analysis of the Sq-variations, May-August 1933, Terrest. Magnetism. Atmospheric Elec., 45, 425–432.

Chapman, S., 1919; The solar and lunar diurnal variation of the earth's magnetism, Phil. Trans. Roy. Soc. London, A, 218, 1–118.

Hasegawa, M. and M. Ota, 1950; On the magnetic field of Sq in the middle and low latitudes during the Second Polar Year, Trans. Oslo Meeting I.A.T.M.E., Bull. No. **13**, 426–430.

Matsushita, S. and H. Maeda, 1965; On the geomagnetic solar quiet daily variation field during the IGY, J. Geophys. Res., **70**, 2535–2558.

Ota, M., 1954; An analysis on the diurnal variation of the terrestrial magnetism, especially on the day-time-variation of geomagnetically quiet days, J. Geomag. Geoele., **6**, 83–98.

Price, A. T. and G. A. Wilkins, 1963; New methods for the analysis of geomagnetic fields and their application to the Sq field of 1932–33, Phil. Trans. Roy. Soc. London, A, **256**, 31–98.

Sugiuray, M. and M. P. Hagan, 1967; Universal-time change in the geomagnetic solar quiet daily variation Sq, Scientific Report, NSF GA-478.